Highly Compressible MHD Turbulence and Gravitational Collapse

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Abstract. We investigate the properties of highly compressible turbulence and its ability to produce self-gravitating structures. The compressibility is parameterized by an effective polytropic exponent γ_e. In the limit of small γ_e, the density jump at shocks is shown to be of the order of e^{M^2}, and the production of vorticity by the nonlinear terms appears to be negligible. In the presence of self-gravity, we suggest that turbulence can produce bound structures for $\gamma_e < 2(1-1/n)$, where n is the typical dimensionality of the turbulent compressions. We show, by means of numerical simulations, that, for sufficiently small γ_e, small-scale turbulent density fluctuations eventually collapse even though the medium is globally stable. This result is preserved in the presence of a magnetic field for supercritical mass-to-flux ratios.

INTRODUCTION

In this paper we present a brief discussion on the dynamical properties of highly compressible turbulence, with and without self-gravity. A full-length discussion can be found in Vázquez-Semadeni *et al.* (1996, hereafter Paper I). The compressibility of the medium is parameterized by an effective polytropic exponent γ_e arising from balance between heating and cooling processes (Elmegreen 1991; Vázquez-Semadeni *et al.* 1995; Passot *et al.* 1995), such that the pressure P and the density ρ are related by $P \propto \rho^{\gamma_e}$. We consider the cases where γ_e is either constant, or has a piecewise density dependence (a "piecewise polytropic model" or *ppm*). We also consider fully thermodynamic cases. Most of the results are based on numerical calculations which solve the full MHD equations in two or three dimensions, at resolutions of 128^2 or 64^3, respectively (Paper I).

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NO SELF-GRAVITY

Density Structures

It can be easily shown that the density jump $X \equiv \rho_2/\rho_1$ across a shock in a barotropic gas of index γ_e satisfies:

$$X^{1+\gamma_\mathrm{e}} - (1+\gamma_\mathrm{e} M^2)X + \gamma_\mathrm{e} M^2 = 0, \tag{1}$$

where M is the upstream Mach number.

In the isothermal case where $\gamma_\mathrm{e} = 1$, $X = M^2$, while for $0 < \gamma_\mathrm{e} \ll 1$, $X \sim e^{M^2}$. This implies that for small γ_e :
i) The density jump is much larger than in the isothermal case.
ii) Less than supersonic motions (with respect to the isothermal sound speed) are sufficient for producing large density fluctuations.

Vorticity evolution

From the momentum conservation equation (Paper I), we can derive the evolution equation for the *potential vorticity* $\omega_\mathrm{p} \equiv \omega/\rho \equiv \nabla \times \mathbf{u}/\rho$:

$$\frac{\partial \omega_\mathrm{p}}{\partial t} + \mathbf{u} \cdot \nabla \omega_\mathrm{p} = \omega_\mathrm{p} \cdot \nabla \mathbf{u} + \nabla \times \mathbf{F}_\mathrm{s} + \nabla \mathbf{P} \times \nabla \rho / \rho^{\mathbf{3}} \tag{2}$$

where $\mathbf{F}_\mathrm{s}$ is the solenoidal (or rotational) part of the turbulent energy sources.

While compressive motions can easily be generated from vortical motions, only the "vortex stretching" term $\omega_\mathrm{p} \cdot \nabla \mathbf{u}$ and the "baroclinic" term (nonzero only in the fully thermodynamic case) are available as nonlinear sources of ω_p. How effective are they?

Fig. 1 (*left*) shows the evolution of the ratio $e_\mathrm{s}/e_\mathrm{k}$ of solenoidal to total kinetic energy per unit mass for four 3D runs. Runs 60 and 67 are *ppm* runs with $\gamma_{\mathrm{e}_\mathrm{min}} = 0.25$ and $\gamma_{\mathrm{e}_\mathrm{min}} = 0.75$, both with fully solenoidal initial velocity modes but fully compressible forcing ($\mathbf{F}_\mathrm{s} \equiv \mathbf{0}$). Run 68 is similar to run 60, but fully thermodynamic. Run 69 is similar to run 60, but with a Coriolis term added. In all cases except run 69, $e_\mathrm{s}/e_\mathrm{k}$ is seen to decay at roughly the same rate in time. Thus :
i) We have found negligible nonlinear transfer from compressible to rotational kinetic energies. Similar results in the weakly compressible case have been found by Kida & Orszag (1991a,b).
ii) Additional sources of vorticity, such as the Coriolis force (acting on large scales), are necessary for the maintenance of significant amounts of vorticity.

Additionally, the presence of an initially uniform magnetic field also allows for the production of rotational energy from the compressive motions, with an efficiency proportional to the magnetic field strength (fig. 1 (*right*) ; in runs 86, 84, 87, 88, the non-dimensional field intensity is respectively 0.05, 0.3, 1 and 3). This process is more important at small scales.

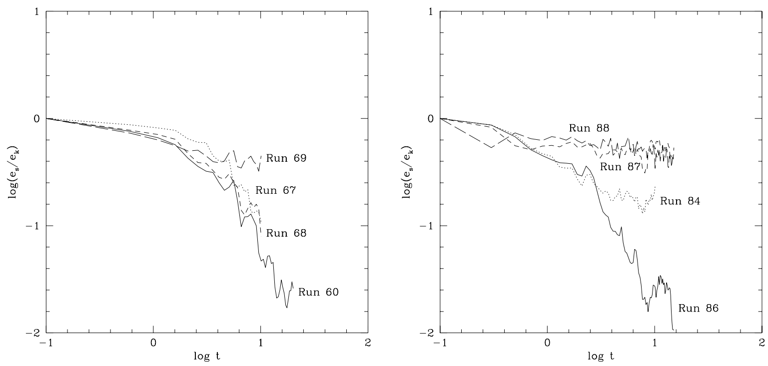

FIGURE 1. Evolution of the ratio of rotational to total specific kinetic energy for various runs. *Left*: non-magnetic runs. *Right*: magnetic runs.

FLOWS WITH SELF-GRAVITY

Non-magnetic case

It can readily be shown that for a barotropic medium with polytropic exponent $\gamma_{\rm e}$, the effective Jeans length is :

$$L_{\rm eff} = \Big[\frac{\gamma_{\rm e}\pi c_{\rm i}^2}{G\rho_{\rm o}^{2-\gamma_{\rm e}}}\Big]^{1/2} = \sqrt{\frac{\gamma_{\rm e}}{\gamma}}\rho^{\frac{\gamma_{\rm e}-1}{2}} L_{\rm J}, \tag{3}$$

where $c_{\rm i}$ is the isothermal sound speed, such that $P = c_{\rm i}^2\rho$, and $L_{\rm J}$ is the Jeans length based on $c_{\rm i}$.

The critical density required for destabilizing a length scale L is thus $\rho_{\rm J} \propto L^{2/(\gamma_{\rm e}-2)}$. In order to account for turbulent compressions acting on n directions, we consider a volume $V = L^n L_0^{3-n}$, where L is the side of the volume which varies upon compression, and L_0 is the side that remains unaltered. The critical mass to destabilize this volume is thus given by $M_{\rm J} \propto L^{\frac{n+2}{\gamma_{\rm e}-2}} L_0^{3-n}$. If $M_{\rm J}$ is a decreasing function of L, turbulent compression can produce gravitationally unstable structures, i.e, if (see also McKee et al. 1993)

$$\gamma_{\rm e} < \gamma_{\rm crit} \equiv 2(1 - 1/n). \tag{4}$$

This result is illustrated in fig. 2 (*left*), which shows the evolution of the global density maximum for three single-$\gamma_{\rm e}$ simulations, all with an isothermal Jeans length $L_{\rm J} = 1.1$ times the box size (i.e., gravitationally stable) and purely compressible random forcing, but with $\gamma_{\rm e} = 0.9$, 0.3 and 0.1 respectively. While the simulation with $\gamma_{\rm e} = 0.9$ never develops a gravitational collapse, the other two do, earlier in the case with $\gamma_{\rm e} = 0.1$.

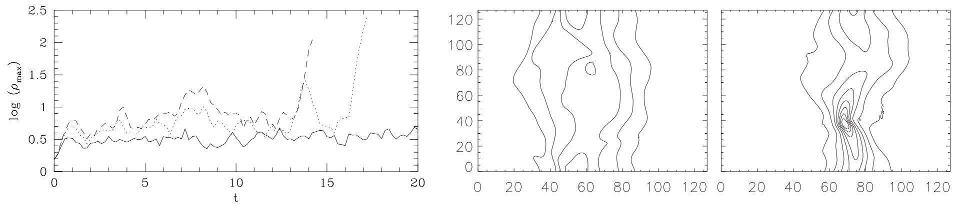

FIGURE 2. *Left*: Evolution of the global density maximum for three runs with $\gamma_e = 0.9$ (*solid line*), $\gamma_e = 0.3$ (*dotted line*) and $\gamma_e = 0.1$ (*dashed line*). *Right*: Final density fields of two magnetic simulations, one with $\gamma_e = 0.9$ (*center*) and one with $\gamma_e = 0.3$ (*right*).

Magnetic case

The above result is preserved in the presence of a magnetic field. Fig. 2 also shows contour plots of the final density field in simulations with an initially uniform magnetic field along the x-direction, a Jeans length $L_J = 0.9$ (i.e., gravitationally unstable with respect to a uniform density configuration), and purely compressible random forcing. One simulation has $\gamma_e = 0.9$ (*center*) and the other has $\gamma_e = 0.3$ (*right*). The latter is seen to have undergone collapse, while the former only contracts to a pancake-type structure supported by thermal pressure against final collapse. However, for large enough magnetic strengths, we have found that turbulence is again not capable of inducing gravitational collapse, suggesting that a subcritical magnetic regime (Mouschovias & Spitzer 1976) cannot be forced to collapse by large-scale turbulence (although the presence of small-scale turbulent modes may affect this result).

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